

NOTES FROM YESTERDAY:

Dot Product:

If $\vec{u} = \langle a_1, a_2, a_3 \rangle$ and $\vec{v} = \langle b_1, b_2, b_3 \rangle$

$$\text{then } \mathbf{u \cdot v} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

If $\mathbf{u \cdot v} = 0$, then vector \mathbf{u} and \mathbf{v} are perpendicular.

NOTES: 9.5

The cross product finds a vector that is perpendicular (orthogonal) to 2 given vectors that are in the same plane.

$$\mathbf{a} \times \mathbf{b} = \text{cross product}$$

↑ *Not a multiplication symbol.
A matrix will be used to
calculate the cross product.*

Example: find a 3rd vector that is perpendicular to the two given vectors.

$$\vec{a} = \langle -2, -3, 1 \rangle$$

$$\vec{b} = \langle 2, 5, -4 \rangle$$

$$\text{so... } \vec{a} \times \vec{b} = \langle ?, ?, ? \rangle$$

Given:

$$\vec{a} = \langle -2, -3, 1 \rangle$$

$$\vec{b} = \langle 2, 5, -4 \rangle$$

1st step: set up a 3 by 3 determinant

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -3 & 1 \\ 2 & 5 & -4 \end{vmatrix}$$

2nd step: evaluate using 2 by 2 *minor* determinants

$$\vec{i} \begin{vmatrix} -3 & 1 \\ 5 & -4 \end{vmatrix} - \vec{j} \begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix} + \vec{k} \begin{vmatrix} -2 & -3 \\ 2 & 5 \end{vmatrix}$$

See notes 10.6 for more details
about determinants!!

**2nd step: evaluate using
2 by 2 minor determinants**

$$\bar{i} \begin{vmatrix} -3 & 1 \\ 5 & -4 \end{vmatrix} - \bar{j} \begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix} + \bar{k} \begin{vmatrix} -2 & -3 \\ 2 & 5 \end{vmatrix}$$

$$(12 - 5)i - (8 - 2)j + (-10 + 6)k$$

$$7i - 6j - 4k = \langle 7, -6, -4 \rangle$$

EQUATION OF A SPHERE

NOTES: 9.3 (part 2)

An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

Example: Center = (5, -2, 13) Radius = 8

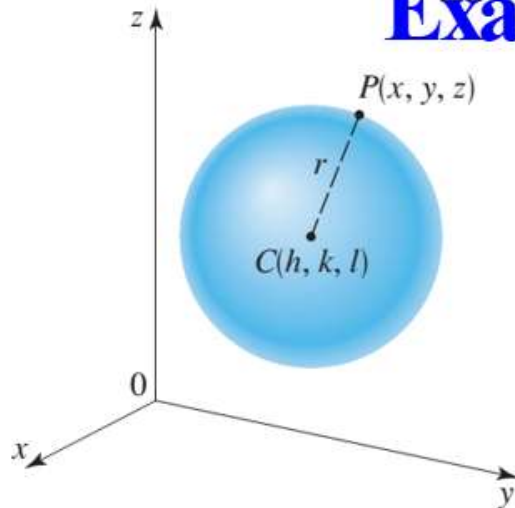


FIGURE 8 Sphere with radius r and center $C(h, k, l)$

Equation of Sphere:

$$(x - 5)^2 + (y + 2)^2 + (z - 13)^2 = 64$$

Assignment: 9.3#18, 11-17 9.5#3,4,7,8,10a,11a

$$x^2 + y^2 - 14y + \underline{49} + z^2 + 6z + \underline{9} = 0 + \underline{49} + \underline{9}$$

Complete the square

$$x^2 + (y-7)^2 + (z+3)^2 = 58$$

$$\text{Center} = (0, 7, -3)$$
$$\text{Radius} = \sqrt{58}$$

15-18 ■ **Center and Radius of a Sphere** Show that the equation represents a sphere, and find its center and radius.

15. $x^2 + y^2 + z^2 - 10x + 2y + 8z = 9$

16. $x^2 + y^2 + z^2 + 4x - 6y + 2z = 10$

17. $x^2 + y^2 + z^2 = 12x + 2y$

18. $x^2 + y^2 + z^2 = 14y - 6z$

Solve by gathering like terms and completing the square.

**9.3 #18, 11-17 and
9.5 #3, 4, 7, 8, 10,11**

√ check odd answers in book

CHECK EVEN ANSWERS:

$$\sqrt{58} \quad 2\sqrt{6} \quad \langle 1, 11, -19 \rangle$$

$$\langle -1, -3, -9 \rangle \quad \langle 7, 1, 4 \rangle$$

$$(-2, 3, -1) \quad (0, 7, -3)$$

$$x^2 + (y-7)^2 + (z+3)^2 = 58$$

$$(x+1)^2 + (y-4)^2 + (z+7)^2 = 9$$

$$(x+2)^2 + (y-3)^2 + (z+1)^2 = 24$$

$$(x+10)^2 + y^2 + (z-1)^2 = 11$$